MATH-SHU 362

Recitation 8: Higher Order ODE

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Exercise 1. *Transform the given equation into a system of first-order equations.*

1. $u'' + \frac{1}{2}u' + 2u = 0.$
2. $t^2u'' + tu' + (t^2 - \frac{1}{4})u = 0.$
3. $u^{(4)} - u = 0.$

Exercise 2. *For the linear system* $\mathbf{x}' = \mathbf{P}(t)\mathbf{x}$, *write down its Wronskian* $W(t)$ *in function of* $W(t_0)$ *and* $\mathbf{P}$.

Exercise 3. *Let* $\mathbf{x}^{(1)}(t), \mathbf{x}^{(2)}(t) \cdots \mathbf{x}^{(n)}(t)$ *be vector functions whose* i*-th components (for some fixed* i*)* $x^{(1)}(t), x^{(2)}(t) \cdots x^{(n)}(t)$ *are linearly independent functions on interval* (a, b). *Conclude that the vector functions are themselves linearly independent on the interval* (a, b).

Exercise 4. *Show that the general solution of* $\mathbf{x}' = \mathbf{P}(t)\mathbf{x} + \mathbf{g}(t)$ *is the sum of any particular solution* $\mathbf{x}^p$ *of this equation and a general solution* $\mathbf{x}^c$ *of the corresponding homogeneous equation.*

Exercise 5. *The Lotka–Volterra equations, also known as the predator–prey equations, are a pair of first-order nonlinear differential equations, frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey. The populations change through time according to the pair of equations:*

$$\begin{cases} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y. \end{cases}$$

Here x *is the number of prey (for example, rabbits);* y *is the number of some predator (for example, foxes) and* $\alpha, \beta, \delta, \gamma$ *are positive parameters. Interpreter this mathematical model in a concrete context.*